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REVEALING OF QUASI-PERIODS IN TIME SERIES BY STRUCTURE LENGTH EMINENCE

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Abstract. We present conceptually simple method for revealing of quasi-periods in time series. The method is based on a function of the structure length eminence, whose maxima positions correspond to quasi-periods (or real periods). The method may be regarded as an analogue of the periodogram analysis, but it works in the real domain dealing with spectrum of periods. The method produces average profile of any specified repeating structure. The method is applicable on time series with constant point step and neglect large scale trend. Otherwise, the time series may be resampled and the large scale trend may be followed and removed by local smoothing. One test of the method is perform on the flickering of the symbiotic star T CrB. System of quasi-periods is revealed.. The basic periods 9.9 min ±8% and 16.3 min. Other test is performed on the prognostic behavior of the atmosphere ^{14}C down to 15 200 years BC. Two basic periods, of 950 yr ± 8% and 2235 yr ± 3% are revealed.

1. INTRODUCTION

Recently we revealed quasi-periods (QPs) in time series (TSs) of the flickering active symbiotic stars RS Oph, T CrB, and MWP 560 (Georgiev et al., 2022). In such cases the frequency methods are difficult to apply, because the TS is short, containing irregular large scale trend and strong noise. We removed preliminary the large scale trend globally, by a polynomial fit over the whole TS. Then we detected QPs by the minima positions of the structure function (SF). Later we resampled the TS within a constant step and detected the same QPs by the maxima positions of the autocorrelation function (ACF). However, these methods derive QPs with a low accuracy and a low resolution, omitting short QPs. By this reason we proposed a method of the structure length eminence (MSLE) (Georgiev, 2023). It checks numerous pixel lengths m as lengths of possible repeating structures and build a function of the structure length eminence (SLE), E(m) , (Eq. 1, 2, Fig.~1e).

Really E(m) is a spectrum of the QPs. Any maximum position of the SLE is a QP, but the real periods may be distinct additionally.

The goals of the present paper are to describe the MSLE and to show two applications. We regard one usual example of star flickering and one complicate example about the prognostic behavior of the ^{14}C in the past. Other examples are discussed in Georgiev (2023).

The used abbreviations follow:
ACF – autocorrelation function;
AV – average value;
MSLE – Method of the structure length eminence;
TS – time series;
QP– quasi-period;
SD – standard deviation;
SF – structure function;
SLE – structure length eminence;
SS – significant (repeating) structure;
SW – smoothing window size.

2. METHOD OF THE STRUCTURE LENGTH EMINENCE

Suppose to have a TS with n equally spaced datapoints ($F(t_i)$ (i=1,n), with zero global average and neglect large scale trend. Such a TS is the residual TS (Fig.1b), derived from the input TS (Fig. 1a) by a large scale trend removal. Suppose that the TS contains a significant (repeating) structure (SS) with an unknown length of m_S pix. We select m_S checking numerous data lengths m_1 m_2, ... , m_S, ..., m_M (e.g. 20--200 points), as follows.

Beginning with m_1 we pull up the first m_1 values of the TS into a (initially empty) set with cell numbers j=1,m_1. Then, we add there the next m_1 values from the TS. Later we add the next m_1, etc. In such a way we perform $k_1=n/m_1$ (integer number) additions. Then for every $_j$-th cell, containing k_1 additions, we derive the average values (AV) $a(j,m_1)$ and standard deviations (SD) $d(j,m_1)$. The program performs this procedure for every m between m_1 and m_M, deriving the relevant m-th profiles of the structure in AVs and SDs.

Latter the program derives the average amplitude $A(m)$ and the average SD $D(m)$ for every m-th profile:

(1) $$A(m)=<|a_j|>_m, \qquad D(m)=<d_j>_m.$$

Note that $A(m)$ gathers the absolute AVs $|a_j|$ and it is the average amplitude of the structure with the length m pix. Also, $D(m)$ is the average noise in this structure.

When a SS with length of m points is present, it produces maximum of $A(m)$ and minimum of D(m) (Georgiev, 2023, Fig. 1e). Therefore, a more sensitive indicator of SSs is the ratio $A(m)/D(m)$. For convenience, we use it multiplied by 100 and call it *structure length eminence*, SLE:

(2) $$E(m)=100 \times A(m)/D(m).$$

Each value of the SLE, $E(m)$, may be regarded as a kind of a signal-to-noise ratio, expressed by percentages in respect with the noise. Really, the local noise continuum of the SLE must be accounted for a correct estimation of the signal-to-noise ratio. The MSLE requires constant pixel step. In many cases the TS may be resampled by a linear interpolation.

The MSLS is applicable on TS without significant large scale trend and with zero global AV. Therefore, a preliminary amplitude decomposition of the input TS F_I into smoothed TS F_S and residual TS, $F_R=F_S - F_I$, is necessary. The subject of the MSLE is the residual TS $\Delta F(t)$, expressed in percentages of the smoothed TS:

(3) $$\Delta F(t) = 100 \times [F_I(t) - F_S(t)] / F_S(t).$$

Using such a residual TS the results of various MSLE applications may be compared easy.

The simple MSLF is applicable when the large scale trend may be fitted and removed by a low degree polynomial (Georgiev, 2023, Fig. 1). Usually the large scale trend is complicate and the polynomial is not enough flexible. *The basic MSLF* is applicable when the large scale trend is complicate, but it may be followed and removed by a sliding average (Figs. 1 and 2 here). The practice shows that the SW must be about 1.5 times larger than the supposed basic period.

3. FLICKERING PERIODS OF T CRB

We studied the flickering of the symbiotic star T CrB in U-band by the methods of the ACF and SF (Georgiev et al. 2021). Figure 1 represents the application of the MSLE on its light curve *u*12. Here the original TS with step of about 1/6 min is resampled to have a step of 1/10 min everywhere and then it is used as an input TS. Such transform performs smoothing of the jagged TS, and decreases the slope of its SF, but it does not influence the period estimation and ensure better describing of the short period profile shapes.

Figure 1(a) shows the input TS in U-band fluxes and the smoothed TS, both in dependence on the monitoring time t_M. Here AV and SD relate with the input TS. N is the number of the data points. Thee input TS contains complicate trend and strong noise.

Figure 1(b) shows the residual TS (Eq. 3), in percentages toward the smooth level, with zero AV and relevant SD. T_M is the monitoring time. SW = 161 pomys = 16.1 min is the window of the average smoothing, shown by the segment W. The edges of the TS are lost and the length of the residual TS is already 818 point. The residual TS in (b) shows 5 humps and a hint of about 17 min period.

Figures 1(c) and 1(d) show the ACF and the SF of the residual TS in (b), with respective steps τ and θ. These functions detect the shortest period p of 9 – 10 min and a systems of QPs like p, $2p$, $3p$, etc., close in the frames of ±10%. The half width ay the half of the humps is about 20%. The basic periods are not clear.

Figures 1(e) shows the SLE (Eq. 2) as a function of the structure time length t_L of the checked structure (with relevant data length m). The range is 4 – 34 min or 40 – 340 pix. A few humps protrude above the continuum of the SLE. Two of them (dots), at P_1 = 9.9 min and P = 16.3 min, seem to be the basic ones.

The revealing of the basic period includes an account of the shapes of the profiles. (See for details Georgiev, 2023, Figs. 1 and 2): (I) These periods have the highest maximums (above the SLE continuum), having also single profile shapes in (g) and (h); (II) They have clear counterparts at $2P_1$, $3P_1$ and $2P$ (asterisks), with double profiles shapes, like $2P_1$ in (i); (III) The basic period P has the expected "harmonic" counterparts at $P/2$, $3P/4$ and $2P/3$ (circles), again with single profiles, like $P/2$ in (f). These QPs are marked by circles; (IV) Here yhe period P_1 does not coincide with the harmonic at $2P/3$, which is observed rarely, therefore is may be regarded as basic period too. By unknown reason its profile of P_1 (g) is far from a sinusoid shape.

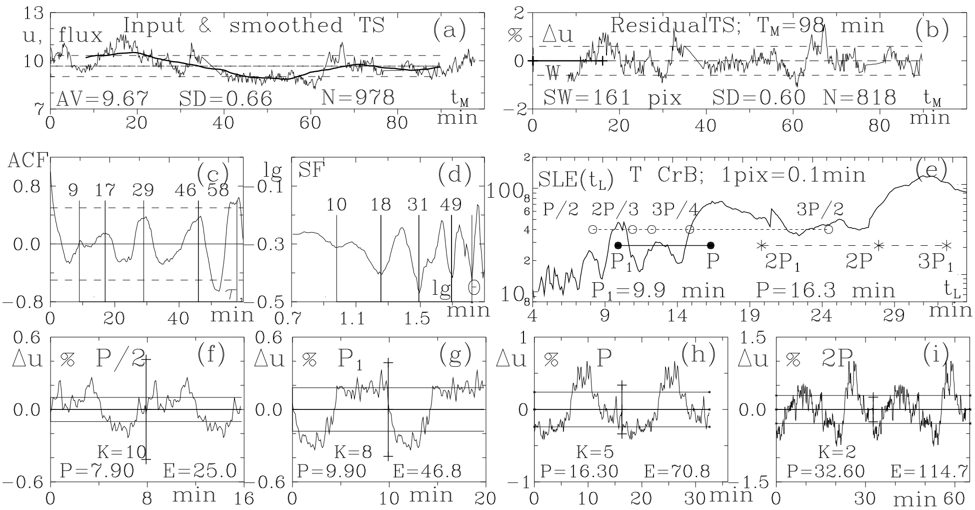

Figure 1: Application of the MSLE on the flickering of T CrB. (a): Input TS and Smoothed TS; (b): Residual TS; (c): ACF; (d): SF; (e): SLE – spectrum of the QPs where only P_1 and $P$are the basic periods; (f – i): Profiles of the SS.

Figures 1(f) – 1(i) show average profiles of SS, taken by the residual TS In (b). In the diagram P is the profile length. There E is the profile eminence, i.e. the absolute height of the relevant maximum in the SLE, without accounting of the SLE continuum. P corresponds to the highest E-value in the relevant hump in the SLE. K is the number of the additions that form the average profile.

So, unlike to the methods of ACF or SF, the MSLE recognizes the basic periods. The half width at the half of the maximum of the P_1-hump is 0.9 cen, i.e.

the external error is 8%. Here the *P*-hump in (e) is not single. Other examples are given in Georgiev (2023). There a comparing with the periodogram analysis of the flickering of AQ Men of Iłkiewich et al. (2021) is included.

4. PERIODS IN THE BEHAVIOUROF THE ^{14}C IN THE FAR PAST

Stuiver et al. (1998) have been published the restored behavior of the relative atmosphere ^{14}C concentration, up to 15 200 years BC. Here the original TS is preliminary resampled to have everywhere 1 step = 1 century and the input TS contains 1520 pix. The input TS (Figs.2(a)) is complicated and noised. The right tail of the TS seems corrupted and only 1325 points are processed by the MSLE.

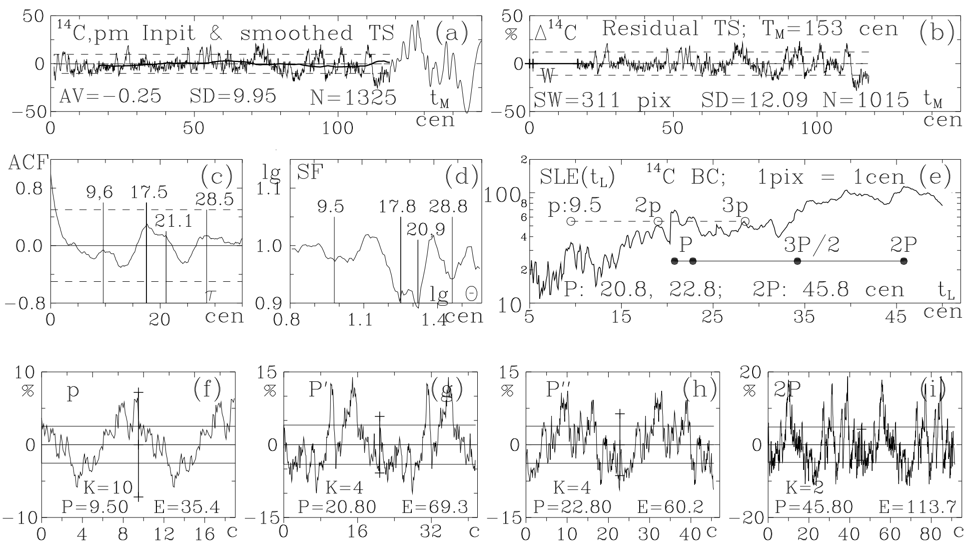

Figure 2: Application of the MSLE on the restored concentration of ^{14}C in the far past. (a): Input TS and smoothed TS in per mille; (b): Residual TS; (c, d): ACF and SF; (e): SLE (the spectrum of the periods); (f – i): Profiles of SS.

Figure 2 illustrates another application of the MSLE, like to the case in Fig. 1. While an eminent period of 23 cen. is suspected to be the basic one, the input TS in (a) is decomposed into smoothed TS and residual TS in (b) by SW size of 311 points. The edges of the TS are lost and the length of the residual TS is already 1015 points. Figures 2(c) and 2(d) show the ACF and the SF. They detect similar systems of QPs, but with low resolution. The basic QP is not clear.

The SLE in Fig. 2(e) is very complicate. The shortest significant period is *p*=9.5 cen. ±8%. The relevant average profile is given in (f). The basic period *P* appears at 20.8 and 22.9 cen., as well as 2*P* at 45.8 cen. The first two periods show single profiles in (g) and (h), respectively. The 2*P* period shows the expected double profile in (i). Obviously the ACF and SF in (c) and (d) recognize

2p + *P* together. The use of the positions of the relevant humps in the MSLE, including 2*P* with double weight, gives the period of the basic cycle to be 22.35 cen. ±3%.

Generally, the Hallstatt Sun cycle of 2200 – 2400 years (Usoskin et al. 2016; Komitov & Kaftan 2003) obviously influences the concentration of the ^{14}C in the atmosphere. The use of the positions of the relevant humps in the MSLE gives the period of the Hallstatt cycle to be 2235 yr.

5. CONCLUSIONS

The MSLE is an analogue of the periodogram analysis, but it works in the real domain. The MSLE is useful and illustrative tool for revealing of QPs and real periods in TS with irregular trend. Four other examples are given in Georgiev (2023).

The SLE is built over the residual TS, with zero global average (Figs. (b)). The MSLE reveals the QPs with high resolution (Figs. (e)) and produces useful profiles of the repeating structures.

The MSLE is applicable when the large scale trend is the obstacle for the period deriving. Then the trend is following and removing by a sliding average with SW size about 1.5 times larger than the supposed large basic period.

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